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# Whole Number Bias of Students in Fraction Number Line Tasks

**Yuan Yuan**[1] · **Kuolong Chen**[2]

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## Abstract
This study used fraction number line estimation tasks to evaluate students' developmental patterns, and the relationship of such tasks with whole number bias was explored. In total, 189 fourth-grade students in a northern Taiwan elementary school were followed over 2 years. The results demonstrated that the students' fraction learning development pattern progressed from being nonlinear to linear, and none of the students' estimations exhibited a logarithmic pattern, as was the case in positive-integer number line estimation tasks. Many of the fourth-grade students (41.3%) did not exhibit a linear pattern of estimation and had clear whole number bias. By the fifth grade, half of these students still had a nonlinear pattern and could not treat fractions as integrated numbers. Instead, they were affected by whole number bias and mistakenly used the denominator to judge the actual fraction value.

**Keywords** Whole number bias · Number line Estimation · Fraction · Elementary school

## Introduction

In the study of mathematical concepts, large differences exist between the basic concepts and the operation rules for integers and fractions. For example, when learning about integers, students know that each number has only one successor (e.g. 4 after 3) and represents a certain value. However, a fraction can be represented in various forms (e.g. equivalent fractions) rather than using one specific set of numbers. Studies on whole number lines have revealed that the ability of students to correctly judge

✉ Yuan Yuan
yuan541020@gmail.com

Kuolong Chen
klchen@mx.nthu.edu.tw

1 Department of Mathematics Education, National Taichung University of Education, Taichung, Taiwan

2 Department of Special Education, National Tsing Hua University, Hsinchu, Taiwan

numbers develops from an immature logarithmic model to a more accurate linear model (Dehaene, 1997; Siegler & Booth, 2004). However, only a few studies have extensively explored the developmental pattern of fraction number lines.

The two integers in a fraction can be confusing for students. When two fractions with the same denominator are summed, only the numerators of the two are summed, and the denominator remains constant. This fraction operation rule is considered challenging for young students who have only just started learning about fractions. Because fractions are learned about after integers, students are easily influenced by their experience of integers and can misuse their integer knowledge while solving problems that involve fractions. Ni and Zhou (2005) used the term "whole number bias" to describe the behavior of inappropriately applying integer knowledge to rational numbers. One of the common errors made by students is that they consider a fraction to be two separate natural numbers rather than one number.

The present study evaluated the learning developmental patterns of students for the learning of fraction estimates on a number line and explored the relationship between such estimates and whole number bias. The research questions were as follows:

(1) Is there a logarithmic model for fraction estimates on a number line?
(2) What is the relationship between fraction estimates on a number line and whole number bias?

## Whole Number Bias in Fraction Comparison

Whole number bias in fractions has been verified in not only elementary and middle-school students but also some college students (DeWolf & Vosniadou, 2011, 2015; Gómez et al., 2014; Meert et al., 2010; Obersteiner et al., 2013; Vamvakoussi et al., 2012; Van Hoof et al., 2017; Van Hoof et al., 2013). In simple comparisons of fractions with the same numerator, researchers observed that students perceived the fraction with the larger denominator as having a greater value (e.g. $\frac{1}{5} > \frac{1}{4}$ because $5>4$). In this scenario, the students demonstrated clear whole number bias when comparing fractions with the same numerator because they considered only one component of the fraction (Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004). The use of fractions with the same denominator or numerator in student evaluations often encourages the students to focus on the integer difference in one component rather than the entire fraction (Huber et al., 2014). Therefore, some researchers have used more complex fractions with different numerators and denominators to examine the phenomenon of whole number bias in students. Fraction comparison problems have been classified into two categories: consistent comparison, or comparison between $\frac{3}{5}$ and $\frac{1}{2}$, where $3>1$ and $5>2$ and hence $\frac{3}{5}$ is greater than $\frac{1}{2}$, and inconsistent comparison, or comparison between $\frac{2}{9}$ and $\frac{1}{4}$, where $2>1$ and $9>4$ but $\frac{2}{9}$ is less than $\frac{1}{4}$. In these studies, the researchers claimed that students with whole number bias tend to make more mistakes in inconsistent comparisons than consistent comparisons, a claim that has been confirmed in multiple studies (DeWolf & Vosniadou, 2011; Fazio et al., 2016; Meert et al., 2010).

Although research has indicated that some students have whole number bias, whether whole number bias gradually disappears with age is controversial. Some researchers believe that whole number bias fades as a student ages because of their longer exposure to fractions, resulting in improved performance on fraction comparison tests. In a study involving a written fraction comparison test of whole number bias by student grade level, Van Hoof et al. (2015) determined that whole number bias did not substantially change as students progressed from fourth and sixth grades. However, as they progressed to higher grades, the students' whole number bias gradually decreased. Students in eighth grade made almost no mistakes in fraction comparisons when only comparing the denominator. In a study on mathematics experts, Obersteiner et al. (2013) observed no differences between the experts in terms of their performance in consistent and inconsistent comparisons, indicating that mathematics experts do not have whole number bias with fractions. Therefore, if students do not master the basic rules of fractions early on in their fraction-learning process, they may become prone to whole number bias, a problem that may last for multiple years. Students who actually learn and master fraction operation rules may still lack conceptual understanding of the magnitude of fractions. Deliyianni et al. (2016) suggested that researchers should provide various representational transformations to examine the representational flexibility of students toward a mathematical concept. According to Michaelidou and Gagatsis (2005), number lines can be used as an assessment tool to trace the misconceptions and difficulties that students face with the concept of fractions. Therefore, in the present study, a number line was used to examine the whole number bias of students.

## Whole Number Bias on a Number Line

Dehaene (1997) proposed the assumption that children comparing two or more numbers actually imagine a number line in their minds, called the mental number line. He further claimed that children compare numbers on the basis of their position on their mental number line. Since these assumptions were proposed, numerous researchers have claimed that being able to accurately estimate the position of a number on a number line is a core ability in the concept of numbers in children. This ability may be a basic component shared by all abilities related to the sense of numbers (Laski & Siegler, 2007; Pan et al., 2009; van de Walle, 1998). This inference has been supported by many researchers, who have considered the use of children's performance in estimating the position of a number on the number line as an effective measuring instrument for the concept of numbers (Berch, 2005; Jordan et al., 2006; Schneider et al., 2008; Siegler & Opfer, 2003). According to previous studies, the estimation performance of students on a number line is related to their mathematical abilities, and their performance may effectively predict their future mathematical abilities (Booth & Siegler, 2006; Halberda & Feigenson, 2008; Siegler & Booth, 2004; Siegler & Opfer, 2003).

According to studies regarding whole number bias on a number line, as students become older and gain more mathematics experience, their ability to correctly judge numbers develops from an immature logarithmic model to a more accurate linear model. In the logarithmic model, a student estimating the positions of positive integers up to 100 on a number line may overestimate the position of a small integer along the line and underestimate the position of a large integer. By contrast, in the linear model, the student makes consistent or identical estimation errors for all integers (Dehaene, 1997; Siegler & Booth, 2004). However, Siegler et al. (2011) suggested that models of fraction estimation using a number line differ from models of positive integer estimation, therefore excluding a logarithmic model. Dehaene (1997) proposed that the logarithmic model is used when students are unfamiliar with the numbers involved. However, students who know the numbers well can make estimations using the linear model. In fraction learning, students may be more familiar with simple fractions—such as $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$—than with more complex fractions, such as $\frac{1}{29}$ and $\frac{28}{29}$. Iuculano and Butterworth (2011) proposed that the fraction learning of students may not necessarily transition from the logarithmic to the linear model. Therefore, sociological experiments are warranted to explore the potential human behavior of comparing fractions by visualizing a number line. Most of the studies on learning development models of number line estimation for students have focused on the average or median data of student groups rather than on individual students. However, measurements of the central tendency in a group overlook the actual performance of individual students. Therefore, student models should be investigated by considering individual student performance.

Because students with whole number bias may not exhibit this problem while using different strategies to solve fraction comparison problems, several researchers have proposed the idea of using number line tasks to identify whole number bias. For example, Braithwaite and Siegler (2018) tested a sample of fourth- and fifth-grade students on their ability with equivalent fractions, which are fractions that have the same value but different numerators and denominators, such as $\frac{3}{5}$ and $\frac{12}{20}$. Students who had whole number bias believed that the fraction with the larger numerator and denominator (in the previous example, $\frac{12}{20}$) had a higher value than the other fraction. Therefore, Braithwaite and Siegler proposed that whole number bias can be detected using equivalent fraction tasks on a number line. In a fraction comparison test, they discovered that the level of whole number bias decreased between the fourth and fifth grade; however, this level remained constant when a number line was instead used. Therefore, they proposed that the level of whole number bias on a number line may not decrease with age and such bias may take a long time to fade.

The fraction comparison test indicates a student's mastery of the related mathematical concepts and their level of whole number bias. However, whether fraction estimates on a number line correlate with whole number bias remains unclear, and this is the topic of the present study.

## Research Method

### Research Participants

This study was conducted over 2 years with fourth-grade students in an elementary school in northern Taiwan. Two tests were conducted for each student: one in fourth grade and the other in fifth grade. Consent was obtained from the students and their parents, and finally 197 students participated. After those who took only one test or did not complete a test were excluded, the final number of participating students was 189 (105 boys and 84 girls). The average age of the participating students was 10 years and 7 months.

### Research Tools

This study used fraction estimates on a number line to investigate the development of learning patterns in students and the effect of whole number bias. The fraction estimate tests used were revised from those in the experimental design of Iuculano and Butterworth (2011). For convenience, tests were performed on a number line with a length of 20 cm (i.e. a line segment on a piece of A4 paper). The right-hand side of the number line was marked with 1, and the left-hand side was marked with 0. Next, a target fraction was provided to the student. The student then estimated the position of the fraction on the number line by marking the line. The target fractions were all less than 1 and were evenly distributed on the number line. To test whether the students' performance was affected by the denominator, the denominators used were all numbers from 2 to 10 and one was 20. Fractions with the same denominator and numerator were also considered in the selection process. The following are the 15 target fractions used and their specific order of appearance: $\frac{4}{9}, \frac{1}{2}, \frac{4}{5}, \frac{3}{4}, \frac{4}{8}, \frac{1}{6}, \frac{1}{3}, \frac{5}{6}, \frac{1}{4}, \frac{8}{10}, \frac{1}{20}, \frac{3}{5}, \frac{5}{8}, \frac{3}{6}$, and $\frac{5}{7}$.

### Research Process

All students took the test in September 2019 and September 2020. The test coordinator demonstrated how the students should mark their answer before the test. No limit was set on the test duration. After the students had completed the test, the test coordinator examined the completeness of their answers. If any question was unanswered, the student was asked to answer it.

### Data Analysis

(1) The students' estimations of fraction values were collected and analyzed. The person performing the analysis used a ruler to measure the distance from the estimated point to 0 on the number line and recorded it to the first decimal place. For example, if a student marked the target fraction $\frac{1}{4}$ at a position with a length of 6.2 cm; then 6.2 was divided by the total length 20. The result obtained (i.e. 0.31) was then used to represent the student's estimated fraction value. Consequently, the absolute value of the difference between the estimated and actual values of the fraction was calculated to obtain the error in the fraction estimate.
(2) The fraction estimate errors of the pooled student sample and for each individual student were analyzed. In the group analysis, the average value of a fraction estimate was calculated for a given target fraction. In the individual analysis, the individual's estimated fraction value for a given target fraction was used.

The regression formulas of the logarithmic and linear models were derived, and the significance of each regression model was used to indicate a learning development pattern (logarithmic or linear). If both models were significant, the regression model with the higher $R^2$ value was selected as the learning pattern. If neither model was significant, the learning development pattern was regarded as unclear.

(3) Finally, the target fractions were arranged in descending order of their denominators. The average estimate of a given fraction made by groups of students exhibiting different patterns was calculated.

## Results

### Learning Development Pattern Revealed by Number Line Fraction Estimates

Group average behavior and individual behavior were analyzed using regression of the fraction estimates of the 189 students in fourth and fifth grades. A statistical significance test was performed on the data to identify reliable patterns of learning development for the group and each individual. In the group analysis, the value of the target fraction was regarded as the independent variable. In the regression analysis, the group average estimates were regarded as the dependent variable. A significant linear relationship was discovered between the exact value and average estimate for both time points (fourth and fifth grade; Figs. 1 and 2).

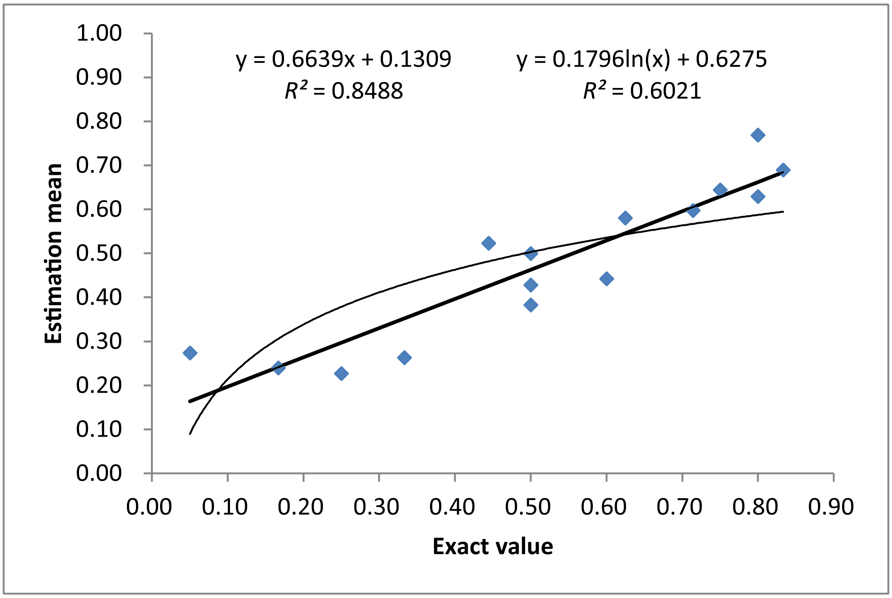


**Fig. 1** Regression analysis of the actual and estimated values of fractions for students in fourth grade

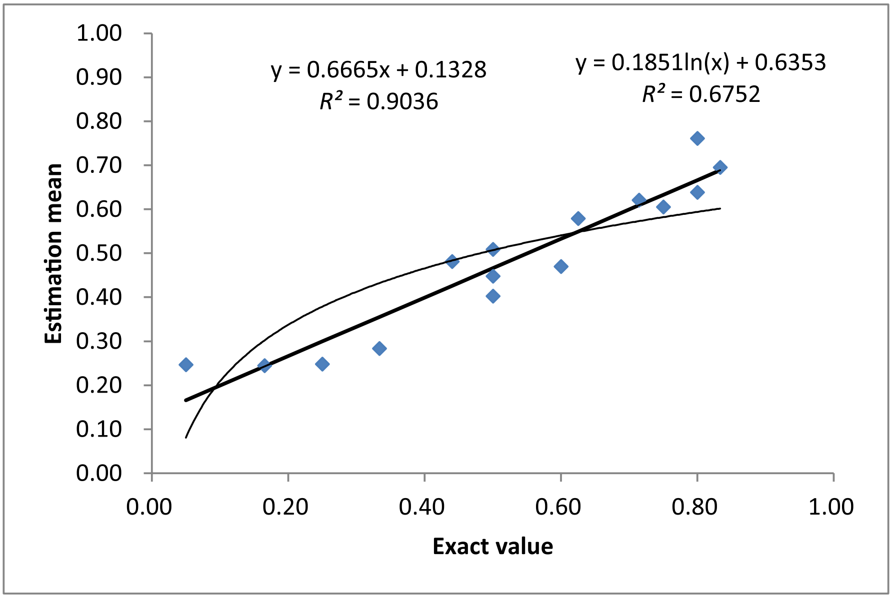


**Fig. 2** Regression analysis of the actual and estimated values of fractions for students in fifth grade

The results obtained from the regression significance test indicated that none of the individual students had an estimation model that exhibited a logarithmic pattern. Among the fourth-grade students, 58.7% had a linear-model learning development pattern, whereas 41.3% had neither a linear nor logarithmic pattern. The results for these students were thus deemed unclear, and their model is referred to as a non-linear model. Once they were in fifth grade, 24.9% of the students had a nonlinear model, whereas 75.1% had a linear model (Table 1).

## Relationship between Number Line Fraction Estimates and Whole Number Bias

We were interested in students whose learning development patterns were stable in grade 4 and grade 5 (i.e., both linear in grader 4 and linear in grade 5). In this section, the two groups of students were further analyzed. Fourth- and fifth-grade students with a nonlinear model were assigned to the nonlinear model group (38 students), whereas those with a linear model were assigned to the linear model group (102 students). The estimated fraction values were compared

**Table 1** Learning development pattern reflected by fraction estimates on the number line for fourth- and fifth-grade students

| | | Grade 5 | | |
|---|---|---|---|---|
| | Model | Nonlinear | Linear | Total |
| Grade 4 | Nonlinear | 38 (20.1%) | 40 (21.2%) | 78 (41.3%) |
| | Linear | 9 (4.8%) | 102 (53.9%) | 111 (58.7%) |
| | Total | 47 (24.9%) | 142 (75.1%) | 189 (100%) |

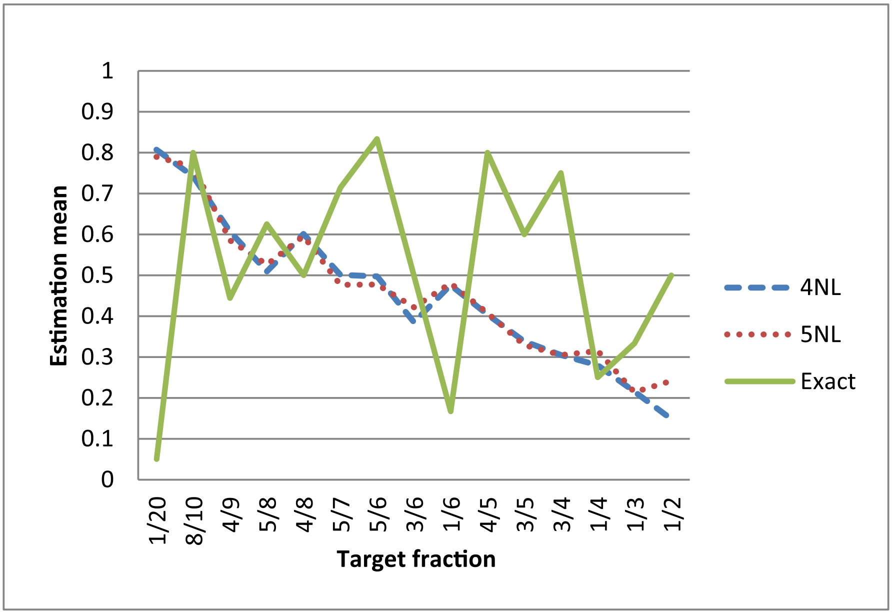


**Fig. 3** Comparison of estimated and actual values of target fractions for the fourth- and fifth-grade nonlinear (NL) group

with the actual fraction values for each group. In the case of whole number bias, the student usually interpreted the fraction with the larger denominator as being the largest in value. A total of 15 target fractions were arranged in descending order of their denominator. Comparisons of the estimated and actual values for the two groups are presented in Figs. 3 and 4. Both the fourth- and fifth-grade nonlinear groups exhibited clear whole number bias; they overestimated fractions with large denominators and underestimated fractions with small denominators. Despite an improvement in estimation of the fraction $\frac{1}{2}$, once in fifth grade, the students in the nonlinear group still exhibited whole number bias (Fig. 3). By contrast, the linear group did not demonstrate clear whole number bias in either grade; their estimated values matched the actual values of the fractions (Figs. 4 and 5).

## Discussion of Integer Bias Affected by Fraction Type

To further understand the concept of whole number bias being affected by different fraction types, the present study analyzed the effects of unit fractions, equivalent fractions, and fractions with the same denominator.

(1) Unit fractions: The following five target fractions were used as unit fractions in the experiment: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{20}$. Comparison of the estimated and actual values for the linear and nonlinear groups of students indicated that the nonlinear group exhibited whole number bias when faced with unit fractions; they

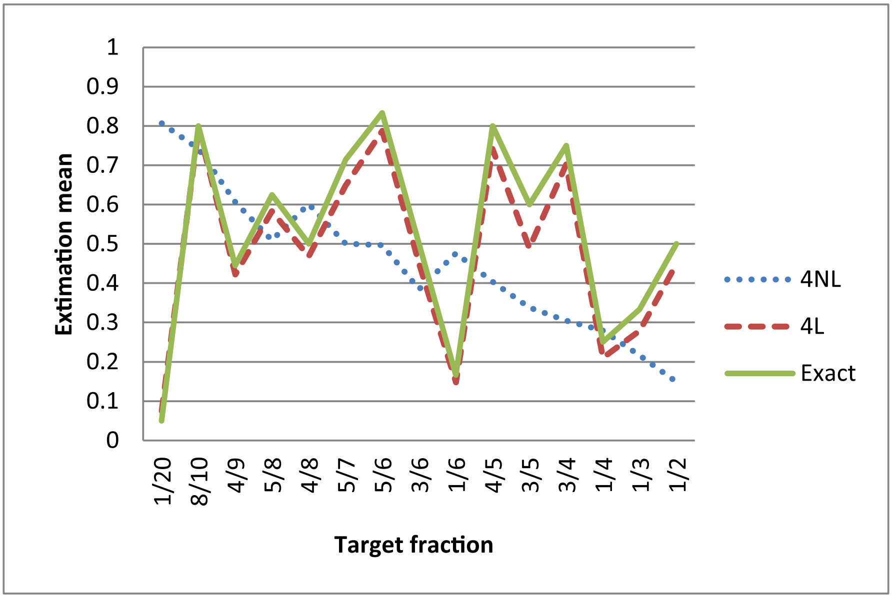


**Fig. 4** Comparison of estimated and actual values of target fractions for the fourth-grade linear (4L) and nonlinear (4NL) groups

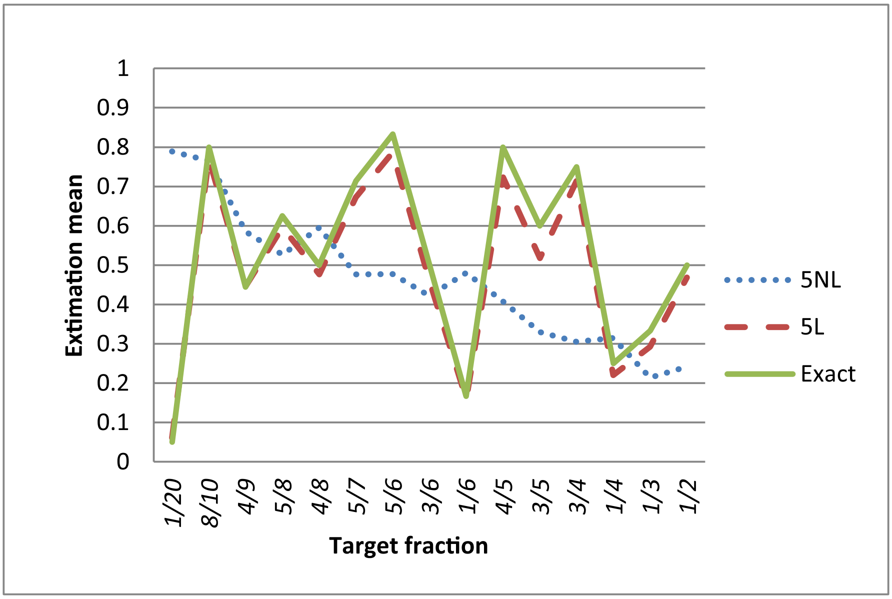


**Fig. 5** Comparison of estimated and actual values of target fractions for fifth-grade linear (5L) and nonlinear (5NL) groups

overestimated fractions with large denominators and underestimated fractions with small denominators. However, no substantial differences were discovered between the estimated and actual values in the linear group (Fig. 6).

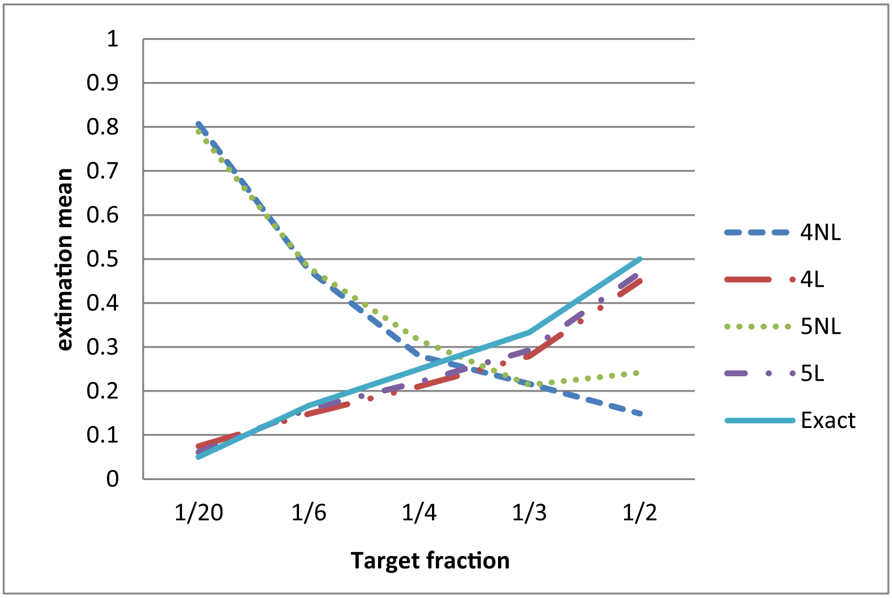


**Fig. 6** Comparison of estimated and actual values of target unit fractions for fourth- and fifth-grade linear (L) and nonlinear (NL) groups

(2) Equivalent fractions: The following two sets of equivalent fractions were used as target fractions in the experiment: $\frac{1}{2}$, $\frac{4}{8}$, and $\frac{3}{6}$(first set) and $\frac{4}{5}$ and $\frac{8}{10}$ (second set). Comparisons of the estimated and actual values for the linear and nonlinear groups revealed consistent results. Again, the nonlinear group exhibited whole number bias when faced with equivalent fractions by overestimating the fractions with large denominators. However, the estimates of the linear group were unaffected by the integer values of the numerator and denominator, resulting in only minor differences between the estimates and actual values (Figs. 7 and 8).

(3) Fractions with the same denominator: The following four sets of fractions with the same denominator were used in the experiment: $\frac{5}{6}$, $\frac{3}{6}$, and $\frac{1}{6}$ (first set); $\frac{4}{5}$ and $\frac{3}{5}$ (second set); $\frac{3}{4}$ and $\frac{1}{4}$ (third set); and $\frac{4}{8}$ and $\frac{5}{8}$ (fourth set). Comparisons between the estimated and actual values of the linear and nonlinear groups indicated that the nonlinear group exhibited less accurate estimates than those of the linear group. However, the trend of higher estimated values due to higher numerator values did not continue, and no correlation was observed between the numerator and the estimated value. Therefore, the performance of the students in the nonlinear group was not affected by the numerators (Figs. 9, 10, 11 and 12).

## Discussion

In this study, number line fraction estimation tasks were used to determine whether the logarithmic model exhibited by elementary school students in their fraction learning is similar to the logarithmic model associated with the learning of

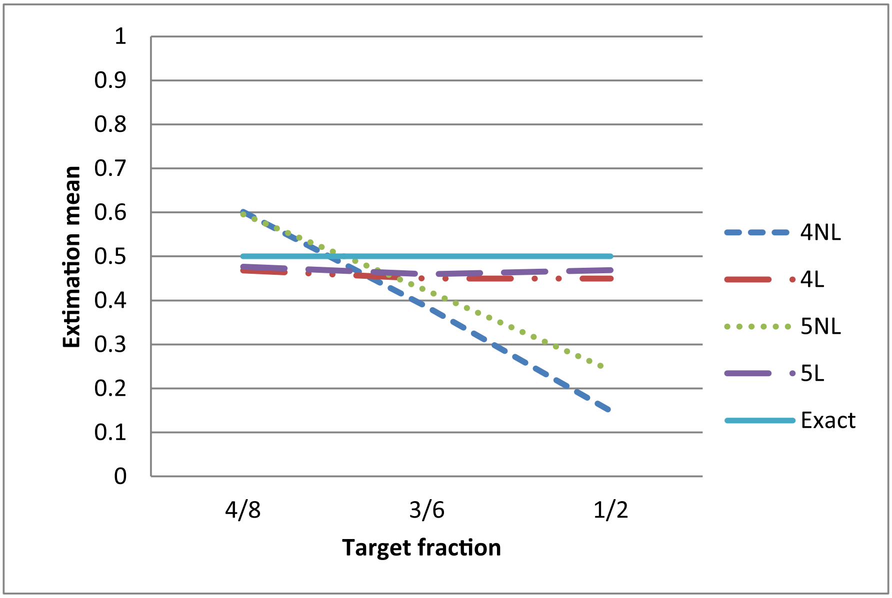


**Fig. 7** Comparison of estimated and actual values of the target equivalent fraction $\frac{1}{2}$ for fourth- and fifth-grade linear (L) and nonlinear (NL) groups

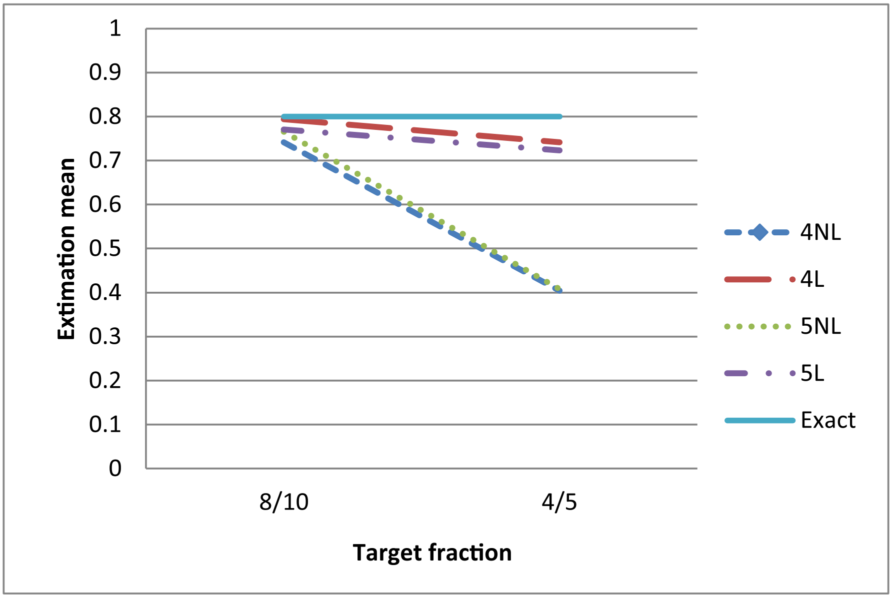


**Fig. 8** Comparison of estimated and actual values of the target equivalent fraction $\frac{4}{5}$ for fourth- and fifth-grade linear (L) and nonlinear (NL) groups

positive integers. When identifying the learning pattern by the group average, the pattern appeared to be linear. The average estimates of the participants indicated the linear model in their fraction value estimation, and this was consistent with

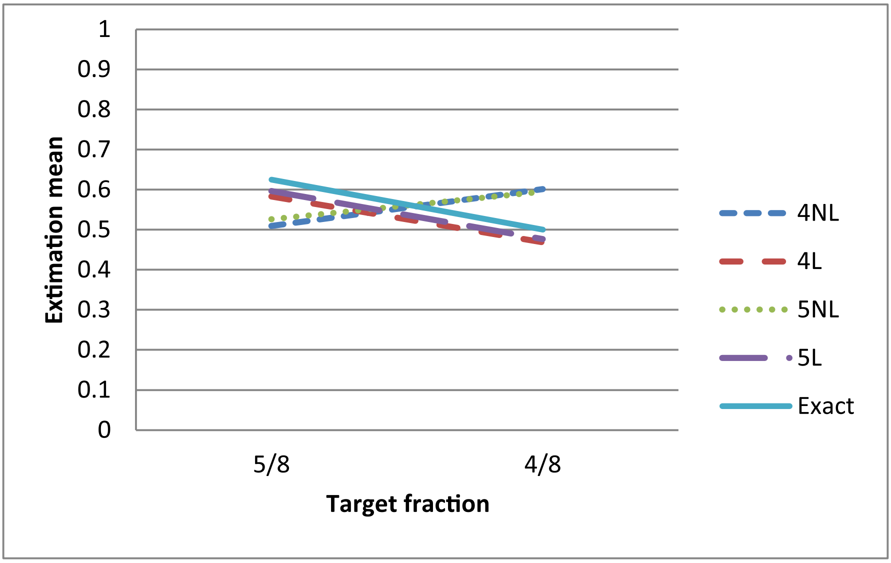


**Fig. 9** Comparison of estimated and actual values of target fractions with the same denominator (i.e. 8) for fourth- and fifth-grade linear (L) and nonlinear (NL) groups

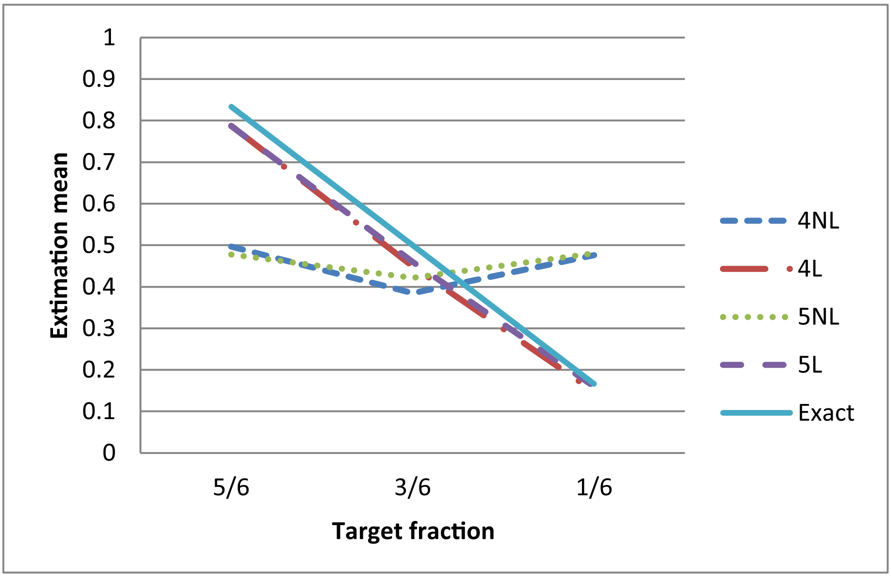


**Fig. 10** Comparison of estimated and actual values of target fractions with the same denominator (i.e. 6) for fourth- and fifth-grade linear (L) and nonlinear (NL) groups

each student's individual estimates. Moreover, many students exhibited a nonlinear pattern that did not indicate a clear learning trend. Siegler et al. (2011) highlighted the absence of a logarithm pattern in fraction estimates. The present findings

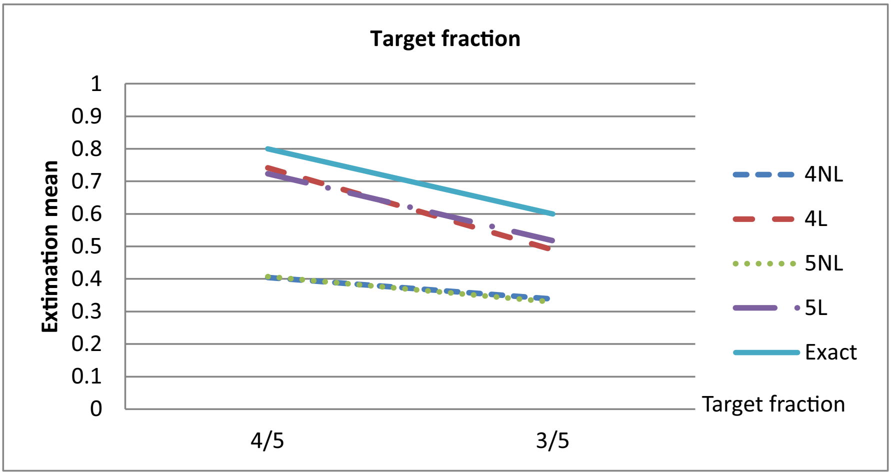


**Fig. 11** Comparison of estimated and actual values of target fractions with the same denominator (i.e. 5) for fourth- and fifth-grade linear (L) and nonlinear (NL) groups

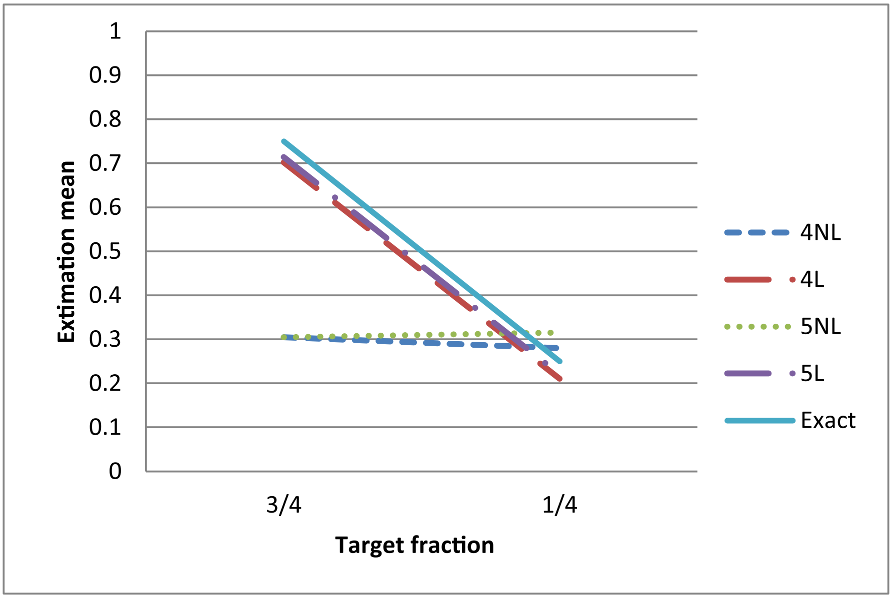


**Fig. 12** Comparison of estimated and actual values of target fractions with the same denominator (i.e. 4) for fourth- and fifth-grade linear (L) and nonlinear (NL) groups

support these results. Although both integers and fractions are numbers, they have different attributes. Finding a fraction on a number line requires cognitive involvement, for example, partitioning and labeling (Klein et al., 1998), which may result in different patterns of development in the two types of number line estimation tasks (integer and fraction estimations). The results obtained also indicated that 41.3% of fourth-grade students adopted a nonlinear model, whereas 21.2% of fifth-grade students had a linear pattern of fraction estimation. Therefore, fraction value estimation develops differently from positive integer estimation. In other words, fraction value estimation develops from a nonlinear to a linear pattern.

Comparison of the linear and nonlinear groups indicated that the estimates of the linear group were close to the actual fraction values and that these groups exhibited minor whole number bias. However, the nonlinear group overestimated the fraction values for fractions with large denominators, thus displaying clear whole number bias. These results support the argument made by Braithwaite and Siegler (2018), who reported that number line fraction estimates can be used to assess whole number bias. However, in contrast to the results obtained by Braithwaite et al., the present study determined that 41.3% of the participants were in the nonlinear group in fourth grade but this percentage decreased to 24.9% in fifth grade, indicating a decrease in whole number bias as the students became older. This discrepancy may have resulted from the methods that Braithwaite et al. used; they did not compare performance in the same students as they aged. Van Hoof et al. (2015) determined that whole number bias in fraction estimation took 4 years to fade, which accords with the findings of Braithwaite et al. However, in the present study, half of the participating students corrected their whole number bias in only 1 year. Compared with their counterparts, these students may have gained more knowledge of fraction concepts from instruction. Mazzocco et al. (2013) proposed that students with low mathematics scores may acquire knowledge from one-half fractions first with a gradual shift to more difficult fractions and that such students may perform as well as typically achieving students after 1 year. They also reported that students with mathematics learning disability students continue to struggle with fractions throughout middle school. This may explain why 20.1% of the students in the present study did not develop a linear pattern of fraction estimation. However, because this study was limited to fourth- and fifth-grade students, the present data cannot be used to predict the time required for students initially in the nonlinear group to eliminate their whole number bias, which is a limitation of this study. Therefore, in future studies, scholars should investigate the conceptual change in students with a nonlinear pattern. Additionally, this study did not explore the characteristic differences between the two groups, which was another limitation. Therefore, in future studies, researchers should collect background information on the students and use this information in further intergroup comparisons.

In this study's unit and equivalent fraction experiments, the nonlinear group clearly exhibited whole number bias. However, in the experiment involving fractions with the same denominator, the students did not estimate an excessively large value when the target fraction had a large numerator, indicating that the fraction estimation behavior of the nonlinear group was mainly influenced by the denominator. This accords with the findings of Braithwaite and Siegler (2018). The present study found that only the nonlinear group exhibited whole number bias in their unit and equivalent fraction estimation. Stafylidou and Vosniadou (2004) indicated that the whole number bias of typically developing students gradually disappears once they understand that fractions are numbers. This indicates that the nonlinear group did not consider fractions as representative of a single number. Instead, they were influenced by the value of the denominator and mistakenly believed that a larger denominator indicates a greater fraction value.

The fact that nine students (4.8%) shifted from a linear to a nonlinear model was relatively surprising. One reason for this is that these students may have been at a stage of conceptual instability. However, explaining this phenomenon requires more information, and further research is warranted.

## Conclusions

This study used fraction number line estimation tasks to evaluate the learning developmental patterns of students and explored the relationship between such tasks and whole number bias. Additionally, the number line fraction estimation performance of individual students was analyzed. The results obtained indicated the absence of a logarithmic pattern in the students' estimation of positive numbers for fractions. Instead, the fraction learning development pattern progressed from a nonlinear to a linear model. Many fourth-grade students (41.3%) did not exhibit a linear pattern of estimation and, therefore, exhibited clear whole number bias. By fifth grade, half of these students still exhibited a nonlinear pattern and could not treat fractions as integrated numbers. Instead, they were affected by whole number bias and mistakenly used the denominator to judge the actual value of a fraction. In conclusion, whole number bias in fraction learning should be scrutinized when teaching fractions to students to develop new effective learning plans that can help students better understand the magnitude of fractions.

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### Declarations

The participants were the entire body of fourth-grade students in a medium-sized elementary school in Zhongli, Taiwan. This study performed data collection of these participants for 2 years. The research obtained the consent of the principals and teachers of the elementary school in the 108th school year (September, 2019), as well as the consent of the parents. For those who choose not to participate in the study, the researcher arranged learning activities at the same test time so that students who have not participated in the research can still participate in learning. In the beginning, an explanation handout was given to the students' parents by each class's homeroom teacher on behalf of the research team, and the study commenced only after obtaining the parents' letters of informed consent. The researcher discussed the test time with the school teacher, and on the principle of not affecting the student's course progress, the test was conducted with the assistance of the school instructor.

**Conflict of Interest** The authors declare no competing interests.

## References

Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38*, 333–339. https://doi.org/10.1177/00222194050380040901

Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology, 42*, 189–201. https://doi.org/10.1037/0012-1649.41.6.189

Braithwaite, D. W., & Siegler, R. S. (2018). Developmental changes in the whole number bias. *Developmental Science, 21*(2), e12541. https://doi.org/10.1111/desc.12541

Dehaene, S. (1997). *The number sense: How the mind create mathematics*. Oxford University Press.

Deliyianni, E., Gagatsis, A., Elia, I., & Panaoura, A. (2016). Representational flexibility and problem-solving ability in fraction and decimal number addition: A structural model. *International Journal of Science and Mathematics Education, 14*, 397–417. https://doi.org/10.1007/s10763-015-9625-6

DeWolf, M., & Vosniadou, S. (2011). The whole number bias in fraction magnitude comparisons in adults. In L. Carlson, C. Hoelscher, & T. F. Shipley (Eds.), *Proceedings of the 33rd Annual Conference of the Cognitive Science Society* (pp. 1751–1756). Cognitive Science Society.

DeWolf, M., & Vosniadou, S. (2015). The representation of fraction magnitudes and the whole number bias reconsidered. *Learning and Instruction, 37*, 39–49. https://doi.org/10.1016/j.learninstruc.2014.07.002

Fazio, L. K., DeWolf, M., & Siegler, R. S. (2016). Strategy use and strategy choice in fraction magnitude comparison. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 42*, 1–16. https://doi.org/10.1037/xlm0000153

Gómez, D., Jiménez, A., Bobadilla, R., Reyes, C., & Dartnell, P. (2014). Exploring fraction comparison in school children. *Enero, 10*, 1–10.

Halberda, J., & Feigenson, L. (2008). Developmental change in acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-old children and adults. *Developmental Psychology, 44*, 1457–1465. https://doi.org/10.1037/a0012682

Hoof, V., Lijnen, T., Verschaffel, L., & Van Dooren, W. (2013). Are secondary school students still hampered by the natural number bias? A reaction time study on fraction comparison tasks. *Research in Mathematics Education, 15*, 154–164. https://doi.org/10.1080/14794802.2013.797747

Huber, S., Moeller, K., & Nuerk, H. C. (2014). Adaptive processing of fractions – evidence from eye-tracking. *Acta Psychologica, 148*, 37–48. https://doi.org/10.1016/j.actpsy.2013.12.010

Iuculano, T., & Butterworth, B. (2011). Rapid communication understanding the real value of fractions and decimals. *The Quarterly Journal of Experimental Psychology, 64*(11), 2088–2098. https://doi.org/10.1080/17470218.2011.604785

Jordan, N. C., Kaplan, D., Olah, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematical difficulties. *Child Development, 77*, 153–175. https://doi.org/10.1111/j.1467-8624.2006.00862.x

Klein, A., Beishuizen, M., & Treffers, A. (1998). The empty number line in Dutch second graders: Realistic versus gradual program design. *Journal for Research in Mathematics Education, 29*(4), 443–464. https://doi.org/10.5951/jresematheduc.29.4.0443

Laski, E. V., & Siegler, R. S. (2007). Is 27 a big number? Correlational and causal connections among numerical categorization, number line estimation, and numerical magnitude comparison. *Child Development, 78*(6), 1723–1743. https://doi.org/10.1111/j.1467-8624.2007.01087.x

Mazzocco, M., Myers, G. F., Lewis, K. E., Hanich, L. B., & Murphy, M. M. (2013). Limited knowledge of fraction representations differentiates middle school students with mathematics learning disability (dyscalculia) versus low mathematics achievement. *Journal of Experimental Child Psychology, 115*, 371–387. https://doi.org/10.1016/j.jecp.2013.01.005

Meert, G. J., Grégoire, J., & Noël, M. P. (2010). Comparing the magnitude of two fractions with common components: Which representations are used by 10-ans 12-year-olds? *Journal of Experimental Child Psychology, 107*, 244–259. https://doi.org/10.1016/j.jecp.2010.04.008

Michaelidou, E., & Gagatsis, A. (2005). The geometrical model of number line as a representation of equivalence and addition of fractions. *Scientia Paedagogica Experimentalis, 42*, 185–204.

Ni, Y., & Zhou, Y. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist, 40*(1), 27–52. https://doi.org/10.1207/s15326985ep4001_3

Obersteiner, A., Van Dooren, W., Van Hoof, J., & Verschaffel, L. (2013). The natural number bias and magnitude representation in fraction comparison by expert mathematicians. *Learning and Instruction, 28*, 64–72. https://doi.org/10.1016/j.learninstruc.2013.05.003

Obersteinera, A., Alibalib, M. W., & Marupudib, V. (2020). Complex fraction comparisons and the natural number bias: The role of benchmarks. *Learning and Instruction, 67*, 1–10. https://doi.org/10.1016/j.learninstruc.2020.101307

Pan, H., Yu, C., & Su, Y. (2009). The development of numerical representation: From the view of number sense. *Journal of East China Normal University, 27*(4), 30–41. (in Chinese).

Schneider, M., Heine, A., Thaler, V., Torbeyns, J., De Smedt, B., Verschaffel, L., Jacobs, A. M., & Stern, E. (2008). A validation of eye movements as a measure of elementary school children's developing number sense. *Cognitive Development, 23*(3), 409–422. https://doi.org/10.1016/j.cogdev.2008.07.002
Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75*, 428–444. https://doi.org/10.1111/j.1467-8624.2004.00684.x
Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations. *Psychological Science, 14*, 237–243.
Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology, 62*(4), 273–296. https://doi.org/10.1016/j.cogpsych.2011.03.001
Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of numerical value of fractions. *Learning and Instruction, 14*(5), 503–518. https://doi.org/10.1016/j.learninstruc.2004.06.015
Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally bias? In search for reaction time evidence for a natural number bias in adults. *The Journal of Mathematical Behavior, 31*, 344–355. https://doi.org/10.1016/j.jmathb.2012.02.001
van de Walle, J. A. (1998). *Elementary and middle school mathematics.* Addison Wesley Longman.
Van Hoof, J., Verschaffel, L., & Van Dooren, W. (2015). Inappropriately applying natural number properties in rational number bias through primary and secondary school. *Educational Studies in Mathematics, 90*, 39–56. https://doi.org/10.1007/s10649-015-9613-3
Van Hoof, J., Verschaffel, L., & Van Dooren, W. (2017). Number sense in the transition from natural to rational numbers. *British Journal of Educational Psychology, 87*(1), 43–56. https://doi.org/10.1111/bjep.12134